2018

MATHEMATICS

(Major)

Paper : 6.1

(Hydrostatics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : $1\times7=7$

(a) Which of the following is a false statement?

Pressure of a fluid at a given depth

(i) is equal in all directions

(ii) depends on the density of the fluid

(iii) acts at right angles to the surface with which it is in contact

(iv) is dependent on the shape of the container

(Choose the correct one)

(b) "Barometers may be used for measurement of altitude of a place."

(Write True or False)

(c) Which of the following statements is true?

For a ship floating in equilibrium, the weight of the ship is

(i) greater than the force of buoyancy

(ii) less than the force of buoyancy

(iii) equal to the force of buoyancy

(iv) independent of the force of buoyancy

(Choose the correct one)

(d) Write an equation that expresses the relation between pressure and volume in adiabatic change.

(e) Define metacentre of floating body.

(f) "The position of the centre of pressure of a plane area is _____ of the inclination of the plane."

(Fill in the blank)

(g) Give an example of application of atmospheric pressure in daily life.

2. Answer the following questions : 2×4=8

(a) State and prove the sufficient condition to be satisfied by a given distribution of forces X, Y, Z so that the fluid may maintain equilibrium.

(b) Obtain the formula for determination of the centre of pressure of any plane area in Cartesian coordinates.

(c) Define surface of buoyancy and surface of floatation.

(d) What do you mean by absolute temperature and absolute zero? What is their scale of measurement?

3. Answer any *three* parts : 5×3=15

(a) If the components parallel to the axes of the forces acting on the element of a fluid at (x, y, z) be proportional to $y^2 + 2\lambda yz + z^2$, $z^2 + 2\mu zx + x^2$, $x^2 + 2\nu xy + y^2$, show that if equilibrium be possible, we must have $2\lambda = 2\mu = 2\nu = 1$.

(b) A tube in the form of a parabola held with its vertex downwards and axis vertical, is filled with different liquids of densities δ and δ'. If the distances of the free surface of the liquids from the focus be r and r' respectively, show that the distance of their common surface from the focus is $\frac{r\delta - r'\delta'}{\delta - \delta'}$.

(c) A semiellipse bounded by its minor axis is just immersed in a liquid, the density of which varies as the depth. If the minor axis be in the surface, find the eccentricity in order that the focus may be the centre of pressure.

(d) If a solid homogeneous right circular cone floats with the axis vertical and vertex downwards, show that the floating is stable if $\frac{\sigma}{\rho} > \cos^6 \alpha$, where σ and ρ are the densities of the solid and the liquid respectively and α is the semivertical angle of the cone.

(e) For atmosphere in convective equilibrium and under constant gravity, show that

$$\frac{T}{T_0} = 1 - \frac{\gamma - 1}{\gamma} \cdot \frac{z}{H}$$

where $\gamma = \frac{C_p}{C_v}$ is constant, p, ρ, T are respectively pressure, density and temperature, H is height of homogeneous atmosphere, T_0 is temperature at sea level where $z = 0$, z is the height of the station.

4. Answer *either (a) or (b)* :

(a) *(i)* A thin sphere of radius a, just filled with water rotates about a vertical diameter with angular velocity $\omega = \sqrt{\frac{2g}{3a}}$; prove that the pressure at any point of the surface of equal pressure which cuts the sphere at right angles is $\frac{4}{3} g\rho a$, ρ being the density of water. 5

(ii) Show that the free surface of a heavy homogeneous liquid at rest under gravity is horizontal. 5

(b) *(i)* A mass m of elastic fluid is rotating about an axis with uniform angular velocity ω, and is acted on by an attraction towards a point on that axis equal to μ times the distance, μ being greater than ω; prove that the equation of a surface of equal density ρ is

$$\mu(x^2 + y^2 + z^2) - \omega^2(x^2 + y^2) = k \log\left\{\frac{\mu(\mu - \omega^2)}{8\pi^3} \cdot \frac{m^2}{\rho^2 k^3}\right\}$$

5

(ii) Find the expression for pressure at any point in an elastic fluid in both the cases, when the temperature remains constant and when the temperature varies. 5

5. Answer *either (a) or (b)* :

(a) *(i)* Prove that the depth of the centre of pressure of a parallelogram, two of whose sides are horizontal and at depths h, k below the surface of a liquid and whose density varies as the depth below the surface is

$$\frac{3(k+h)(k^2+h^2)}{4(k^2+kh+h^2)}$$ 5

(ii) How can you determine the pressure on a curved surface? Find the vertical component of thrust on a curved surface. 5

(b) *(i)* A quadrant of a circle is just immersed vertically with one edge in the surface, in a liquid, the density of which varies as the depth. Determine the centre of pressure. 5

(ii) A vessel full of water is in the form of an eighth part of an ellipsoid (axes a, b, c) bounded by the three principal planes. The axis c is vertical and the atmospheric pressure is neglected. Prove that the resultant fluid pressure on the curved surface is a force of intensity

$$\frac{1}{3}\rho g[b^2c^4+a^2c^4+\frac{1}{4}\pi^2a^2b^2c^2]^{1/2}$$ 5

6. Answer *either (a) or (b)* :

(a) (i) A thin metal circular cylinder contains water to a depth h and floats in water with its axis vertical immersed to a depth h'. Show that the vertical position is stable, if the height of the centre of gravity of the cylinder above its base is less than $\frac{1}{2}(h+h')$. 5

(ii) n volumes $v_1, v_2, ..., v_n$ of different gases at pressures $p_1, p_2, ..., p_n$ and absolute temperatures $T_1, T_2, ... T_n$ are mixed together so that the volume of the mixture is U and the absolute temperature T. Show that the pressure of the mixture is

$$\frac{T}{U}\left(\frac{p_1 v_1}{T_1}+\frac{p_2 v_2}{T_2}+\cdots+\frac{p_n v_n}{T_n}\right)$$ 5

(b) (i) Define metacentric height and obtain an expression for it. 5

(ii) A piston of weight w rests to a vertical cylinder of transverse section k, being supported by a depth a of air. The piston rod receives a vertical blow P, which forces the piston down through a distance h. Prove that

$$(w+\pi k)\left[h+a\log\left(1-\frac{h}{a}\right)\right]+\frac{gP^2}{2x}=0$$

π being the atmospheric pressure. 5

Or

A bent tube of uniform bore, the arms of which are at right angles, revolves with constant angular velocity ω about the axis of one of its arms, which is vertical and has its extremity immersed in water. Prove that the height to which the water will rise in the vertical arm is

$$\frac{\pi}{g\rho}\left(1-e^{-\frac{\omega^2 a^2}{2k}}\right)$$

a being the length of the horizontal arm, π the atomospheric pressure, ρ the density of water and k the ratio of the pressure of the atmosphere to its density.

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